INVESTIGATION OF THE UNSTEADY OPERATION OF A COOLED THERMOELEMENT WITHOUT HEAT REMOVAL FROM THE HOT JUNCTIONS

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The authors present results of a theoretical and experimental study of the unsteady regime of operation of a cooled thermoelement without heat removal from the hot junctions.

In a number of cases of thermostatic control of cooling of objects, only a short duration cycle of operation of thermoelectric batteries is required. For this purpose it is very promising to use unsteady thermoelectric cooling, since heat removal from the hot junctions is not required in the construction of the thermobatteries. It is therefore of interest to study limiting conditions of operation of such batteries, their inertia, and other thermophysical and electrophysical questions.

1. In a mathematical formulation of the problem, we postulate that the geometric dimensions and the physical parameters of the branches are the same, and their side surfaces are adiabatically insulated. We take account of contact electrical resistances, as a rule, by introducing a parameter which is the same for the hot and the cold junctions. With these assumptions, the unsteady temperature field in an unheated thermoelement can be found by solving the differential equation

$$\frac{\partial\theta(X,\mathrm{Fo})}{\partial\mathrm{Fo}}=\frac{\partial^2\theta(X,\ \mathrm{Fo})}{\partial X^2}+\frac{\nu^2}{ZT_0} \tag{1}$$

with the following initial and boundary conditions:

$$\theta(X,\ 0)=1,\quad \left[\frac{\partial\theta}{\partial X}-\nu\theta+\frac{\nu^2 r}{ZT_0}\right]\Big|_{X=0}=0,$$
$$\left[\frac{\partial\theta}{\partial X}-\nu\theta-\frac{\nu^2 r}{ZT_0}\right]\Big|_{X=1}=0, \tag{2}$$

where

$$\theta=\frac{T}{T_0};\quad X=\frac{x}{l};\quad \mathrm{Fo}=\frac{a\tau}{l^2};\quad \nu=\frac{\alpha jl}{\varkappa};\quad ZT_0=\frac{\alpha^2T_0}{\varkappa\rho};\ r=\frac{\rho_k}{\rho l}. \tag{3}$$

To solve this problem in the case of a dc impulse it is convenient to use the operator method of [1]. The final temperature distribution along the branches of the thermoelement is determined by the expression

$$\theta(X,\ \mathrm{Fo})=\frac{2\exp(\nu X+\nu^2\mathrm{Fo})}{\exp\nu+1}\left[\frac{1-\exp(-\nu^2\mathrm{Fo})}{ZT_0}\left(1+r\nu\frac{\exp\nu+1}{\exp\nu-1}\right)+1\right]+$$
$$+\frac{4r\nu^2}{ZT_0}\sum_{m=1}^{\infty}\frac{[1-\exp(-\mathrm{Fo}(2\pi m)^2)]\left(\cos 2\pi mX+\dfrac{\nu\sin 2\pi mX}{2\pi m}\right)}{\nu^2+(2\pi m)^2}+$$
$$+4\nu\sum_{m=1}^{\infty}\frac{\cos\pi(2m-1)X+\dfrac{\nu\sin\pi(2m-1)X}{\pi(2m-1)}}{\nu^2+\pi^2(2m-1)^2}\left\{\frac{\nu^2[1-\exp(-\mathrm{Fo}\,\pi^2(2m-1)^2)]}{ZT_0\pi^2(2m-1)^2}+\exp(-\mathrm{Fo}\,\pi^2(2m-1)^2)\right\}. \tag{4}$$

From Eq. (4) for the cold junction temperature we have

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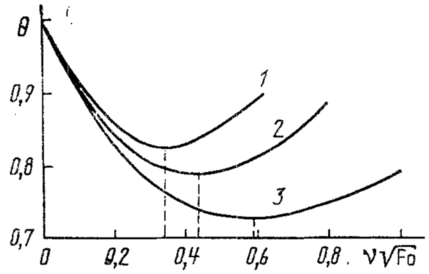

Fig. 1. The cold junction temperature as a function of the parameter $\nu\sqrt{Fo}$ for r = 0: 1) ZT_0 = 0.75; 2) 1.0; 3) 1.5.

$$\theta(0,\ \mathrm{Fo}) = \frac{2\exp(\nu^2\mathrm{Fo})}{\exp\nu+1}\left[\frac{1-\exp(-\nu^2\mathrm{Fo})}{ZT_0}\left(1+r\nu\frac{\exp\nu+1}{\exp\nu-1}\right)+1\right]+ \frac{4r\nu^2}{ZT_0}\sum_{m=1}^{\infty}\frac{1-\exp(-\mathrm{Fo}(2\pi m)^2)}{\nu^2+(2\pi m)^2}+ \frac{4\nu^3}{ZT_0}\sum_{m=1}^{\infty}\frac{1-\exp(-\mathrm{Fo}\,\pi^2(2m-1)^2)}{[\nu^2+\pi^2(2m-1)^2]\,\pi^2(2m-1)^2}+4\nu\sum_{m=1}^{\infty}\frac{\exp(-\mathrm{Fo}\,\pi^2(2m-1)^2)}{\nu^2+\pi^2(2m-1)^2}. \tag{5}$$

An analogous formula is obtained also for the hot junction temperature θ(1, Fo). It is not difficult to see that the first three terms of Eq. (5) increase monotonically with time, while the last one decreases. Therefore, the cold junction temperature is always a minimum while the amount of joule heat liberated at the contact does not exceed the amount of Peltier heat absorbed, i.e., $\nu < ZT_0/r$. For large time intervals the average thermoelement temperature increases exponentially, and for each current value we have the equality

$$\theta(1,\ \mathrm{Fo}) = \exp\nu\,\theta(0,\ \mathrm{Fo}), \tag{6}$$

relating the temperatures of the hot and cold junctions, which can be used, along with the well-known Harman method [2] to determine the thermal conductivity and the thermoelectric efficiency Z.

2. In the analysis of the solutions obtained, it is convenient to consider separately the regions of low and high currents. For the low-current regions we have ν < 1, which corresponds roughly to j_0 or less currents in the steady regime, we can find the Fourier number for which the cold junction temperature at a given time instant is a minimum:

$$\mathrm{Fo}_m = \frac{1}{\pi^2}\ln\frac{4ZT_0}{(1+2r+ZT_0)\,\nu}. \tag{7}$$

Hence it can be seen that as the contact resistance increases there is a decrease, not only in the maximum cooling $1 - \theta_m(0,\ \mathrm{Fo}_m) = \Delta T_m/T_0$, but also in the time at which it is reached.

However, there is much more interest in the high-current region ν > 1, since in this region lies the absolute minimum temperature $\theta_m(0,\ \mathrm{Fo}_m)$. We should transform the solution of Eq. (5) to another more convenient form, especially for short time intervals:

$$\theta(0,\ \mathrm{Fo}) = \left\{\left(1+\frac{1}{ZT_0}\right)\exp(\nu^2\mathrm{Fo})\,\mathrm{erfc}(\nu\sqrt{\mathrm{Fo}})+\frac{2\nu\sqrt{\mathrm{Fo}}}{ZT_0\sqrt{\pi}}-\frac{1}{ZT_0}+ \frac{r\nu}{ZT_0}\left(1-\exp(\nu^2\mathrm{Fo})\,\mathrm{erfc}(\nu\sqrt{\mathrm{Fo}})\right)\right\}+\frac{\nu r}{ZT_0}\exp(\nu^2\mathrm{Fo})\sum_{n=1}^{\infty}\beta_n- \left(1+\frac{1}{ZT_0}\right)\exp(\nu^2\mathrm{Fo})\sum_{n=1}^{\infty}(-1)^n\beta_n+ \frac{2\nu}{ZT_0}\sum_{n=1}^{\infty}(-1)^n\left\{2\sqrt{\frac{\mathrm{Fo}}{\pi}}\exp\left(-\frac{n^2}{4\mathrm{Fo}}\right)-n\,\mathrm{erfc}\left(\frac{n}{2\sqrt{\mathrm{Fo}}}\right)\right\}, \tag{8}$$

where $\beta_n = \exp(-\nu n)\,\mathrm{erfc}\left(-\nu\sqrt{\mathrm{Fo}}+\frac{n}{2\sqrt{\mathrm{Fo}}}\right) - \exp(\nu n)\,\mathrm{erfc}\left(\nu\sqrt{\mathrm{Fo}}+\frac{n}{2\sqrt{\mathrm{Fo}}}\right)$.

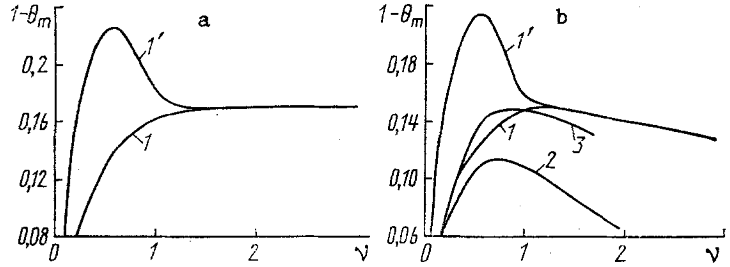

Fig. 2. The maximum variation of the cold junction temperature as a function of current for $ZT_0 = 0.75$; a) r = 0 [1) without heat removal, 1') with heat removed from the hot junction]; b) 1, 1') r = 0.04; 2) 0.16; 3) experiment.

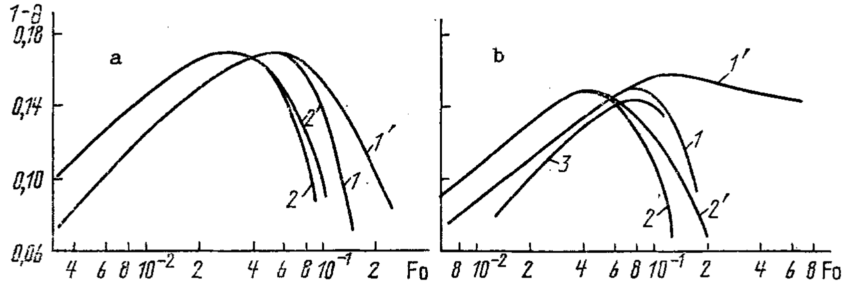

Fig. 3. The variation of cold junction temperature as a function of time for $ZT_0 = 0.75$ with heat conduction (1', 2') and without heat removal from the hot junction (1, 2); a) r = 0 [1, 1') $\nu = 1.5$; 2, 2') $\nu = 2$]; b) r = 0.04 [1, 1') $\nu = 1$; 2, 2') $\nu = 1.5$]; 3) experiment.

The first term in the shaped brackets does not depend on the length of the thermoelement branch and is the exact solution of the problem for a semiinfinite branch (the half space model) [3, 4]. The half-space model is a good approximation for short times $Fo \leqslant 0.05$. Then with sufficient accuracy we can neglect all the other terms in Eq. (8). For the cooling to occur in accordance with the half-space model we must also impose a restriction on the current magnitude. It can be seen from Eq. (8) that for r = 0 $\theta(0, Fo)$ does not depend separately on the current and the time, but is a function of the parameter $\nu\sqrt{Fo}$ (Fig. 1). For $ZT_0=0.75$ the maximum possible cooling $\Delta T_m/T_0$ is 17%, and is reached for different values of ν and Fo, interconnected by the condition $\nu\sqrt{Fo} \simeq 0.34$. This makes it possible to control the cooling rate by choosing suitable values of the current, under the condition $\nu \gtrsim 1.5$. With an increase of thermoelectric efficiency the restriction on the current value shifts to the high-current region. This picture of the variation of $\theta(0, Fo)$ with time and current is perturbed somewhat when one calculates the contact resistance. The cold junction temperature has an absolute minimum for strictly defined Fo_m and ν_m for given r and ZT_0, but for an r not too large this minimum is gently sloping with respect to current, provided that $\nu\sqrt{Fo} = const$.

Figure 2 shows the computer calculated dependence of the maximum variation of cold junction temperature $\Delta T_m/T_0$ as a function of current. For comparison, Fig. 2 also shows the corresponding dependence for a thermoelement operating in the regime with heat removal from the hot junctions. It can be seen that for the first operating regime $\Delta T_m/T_0$ either increases monotonically, tending to the constant value (r = 0), or passes through a gently sloping maximum (r = 0.04), while for the second regime there is sharp maximum for the optimal current:

$$\nu_0 = \sqrt{1 + \frac{2ZT_0}{1+2r}} - 1,$$

when $\Delta T_m/T_0 = \nu_0/(\nu_0 + 2)$.

For multiples of the current $\nu/\nu_0 \geqslant 2.2$, the quantity $\Delta T_m/T_0$ coincides for the two regimes, as must be the case for the half space criterion to hold. The same conclusion is confirmed also by the timewise characteristics of the cold junction temperature drop (Fig. 3).

We now turn briefly to the influence of the contact resistance on the main parameters characterizing the operation of a thermoelement without heat removal. As has been shown above, the quantity Fo_m decreases with increase of r, and for currents larger than j_0 this decrease is considerable. For example, for $ZT_0 = 0.75$ and $\nu = 1.6$ the value of Fo_m decreases from 0.044 at r = 0.01 to 0.023 at r = 0.16. The current ν_m also decreases analogously. For these same values of the quality of the material and the contact resistance, the quantity ν_m falls from 1.42 to 0.8. As regards the maximum cooling $\Delta T_m/T_0$ as a function of the quantity r, one should direct attention to two factors. Firstly, with increase of r, for any deviation of the current from the optimal value the quantity $\Delta T_m/T_0$ falls more sharply (see Fig. 2). Secondly, the value of this maximum also decreases (in our case from 0.164 at r = 0.01 to 0.114 at r = 0.16).

3. The experimental investigations were conducted on thermoelements with branches of prism form of height $15 \cdot 10^{-3}$ m and section area $9 \cdot 10^{-6}$ m^2. As the material of the p and n branches, we used large-crystal specimens of pseudobinary systems based on the chalcogenides of bismuth and antimony with a thermoelectric efficiency of $Z = 2.6 \cdot 10^{-3}$ K^{-1}. The measurements were made in vacuum with a residual pressure not exceeding 10^{-2} Pa. The junction temperatures were monitored with Chromel–Copel thermocouples. The initial thermoelement temperature was $T_0 = 295°K$. As a control we first conducted tests of thermoelements under steady conditions. Figure 2b shows the cold junction temperature as a function of current for $ZT_0 = 0.75$ (curve 3). It can be seen that the curve agrees well qualitatively with the calculations. It has a characteristic maximum, followed by a slow fall. In the increasing section the maximum decrease of cold junction temperature varies with current somewhat more rapidly than it should from the calculations at r = 0.04. We did not measure the contact resistances of the cold and hot junctions, but from the behavior of the curve we can conclude that it is less than 4%.

The decrease of $\Delta T_m/T_0$ in comparison with the expected value is evidently due to factors not accounted for in the calculations, such as heat transfer through the side surfaces, residual convection, and heat leakage through the inlet leads. Figure 3b shows the timewise dependence of the cold junction temperature for $\nu = 0.8$. Because of the danger of breakdown of contacts, we had to limit the measurements for small time intervals. Curve 3 in Fig. 3b agrees well with the calculations.

NOTATION

T, temperature; τ, time; x, coordinate reckoned from the cold junction; a, thermal diffusivity; ℓ, length of the thermoelement branch; α, ρ, and κ, coefficients of the thermal emf, the specific resistance, and thermal conductivity, respectively; ρ_k, specific contact resistance; j, current density; j_0, density of the optimal current in the steady regime; Z, thermoelectric efficiency.

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